

Discrete R-symmetries and F-term Supersymmetry Breaking

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Abstract

We have shown that in a large number of generic and renormalizable Wess-Zumino models existence of a Z_n R-symmetry is sufficient to break supersymmetry spontaneously. This implies that existence of a Z_n R-symmetry is a necessary condition for supersymmetry breaking in generic and renormalizable Wess-Zumino models.

1 Introduction

In discussions of F-term $N = 1$ supersymmetry (SUSY) breaking, R-symmetry plays a crucial role. Importance of a U(1) R-symmetry in spontaneous SUSY breaking was clearly addressed in Ref. [1]. It was shown that, “a continuous R-symmetry is a necessary condition for spontaneous supersymmetry breaking and a spontaneously broken R-symmetry is a sufficient condition, in models where the gauge dynamics can be integrated out and in which the effective superpotential is a generic function consistent with the symmetries of the theory.”

However, in this paper, we are going to show that there exist a large number of generic and renormalizable Wess-Zumino(WZ) models where existence of a Z_n R-symmetry is sufficient to break SUSY spontaneously. It is well known that if there is no R-symmetry, continuous or discrete, in WZ models with canonical Kähler potentials, then the global minima preserve SUSY [1, 2]. With the help of the above two results we can conclude that existence of a Z_n R-symmetry is a necessary condition for breaking in a global minimum for generic and renormalizable WZ models.

Models of F-term SUSY breaking with a Z_n R-symmetry can be obtained most simply from the models of F-term SUSY breaking with U(1) R-symmetry by adding a completely different and decoupled sector to break U(1) R-symmetry explicitly down to a Z_n R-symmetry. In sec 2, we illustrate this idea by adding some more terms to the famous O’Raifeartaigh [3] (O’R) model. However these models are trivial. In sec. 3, we explicitly discuss a non-trivial model where the superpotential cannot be broken into such two non-interacting parts. We have also shown that this model has a vacuum where SUSY as well as discrete R-symmetry is spontaneously broken for large regions of parameter space. We then give some variations of this

model by adding more fields. In sec. 4, we identify the form of these models, and then, three series of models are given where each series contains a large number of such models.

A note about notations. For U(1) R-symmetry we use $R(\phi)$ to denote the U(1) R-charge of any chiral scalar superfield ϕ in the normalization where the R-charge of θ is 1. We use $R_d(\theta)$ to denote Z_n R-charge of the superspace co-ordinate θ . Discrete R-charge of pseudo-moduli superfield X is $2R_d(\theta) \bmod n$ and that of any other superfield is just the subscript of that field.

2 Some trivial examples

Adding a completely different and decoupled sector to any one of F-term SUSY breaking models with U(1) R-symmetry (we will call it as old sector), we can break U(1) R-symmetry to a Z_n R-symmetry explicitly. Value of n is controlled by the new sector. If we find a new sector (or in other words, a Z_n R-symmetry) for which no new term to the old sector is allowed even though R-symmetry becomes weaker, then SUSY breaking conditions coming from the old sector will not alter. In this way, we get models of F-term SUSY breaking with a discrete R-symmetry in generic theories.

To illustrate our idea, we consider the famous O'R model as an example of the old sector:

$$W = fX + a\phi_0\phi_2 + \frac{1}{2}b\phi_0^2X. \quad (1)$$

The above superpotential is generic with a U(1) R-symmetry where $R(X) = R(\phi_2) = 2$, $R(\phi_0) = 0$ and a Z_2 internal symmetry under which X transforms trivially whereas ϕ 's transform non-trivially. Let's now add five new fields $\phi'_0, \phi'_1, \phi'_2, \phi'_3, \phi'_4$, to the old sector and get the following superpotential.

$$\begin{aligned} W = & fX + a\phi_0\phi_2 + \frac{1}{2}b\phi_0^2X + \frac{1}{2}\lambda'_{002}\phi_0'^2\phi_2' + \lambda'_{034}\phi_0'\phi_3'\phi_4' + \frac{1}{2}\lambda'_{011}\phi_0'\phi_1'^2 \\ & + \frac{1}{6}\lambda'_{444}\phi_4'^3 + \lambda'_{124}\phi_1'\phi_2'\phi_4' + \frac{1}{2}\lambda'_{133}\phi_1'\phi_3'^2 + \frac{1}{2}\lambda'_{223}\phi_2'^2\phi_3'. \end{aligned} \quad (2)$$

This superpotential does not have a U(1) R-symmetry, as can be easily checked. However, spontaneous SUSY breaking still occurs because the F-terms, F_X and F_{ϕ_2} , are not changed due to the inclusion of the new terms. The above superpotential is generic with the following three symmetries.

1. A Z_5 R-symmetry with $R_d(\theta) = 1$.
2. A Z_2 internal symmetry under which all the ϕ_i 's transform non-trivially whereas X and ϕ'_α 's transform trivially.
3. A Z_3 internal symmetry under which all the ϕ'_α 's transform as $\phi'_\alpha \rightarrow \omega\phi'_\alpha$ whereas the remaining fields are invariant.

We can get different variations of the above model easily, for a Z_n R-symmetry with $n \geq 5$ and $R_d(\theta) = 1$ as follows.

$$W = fX + a\phi_0\phi_2 + \frac{1}{2}b\phi_0^2X + \frac{1}{6}\lambda'_{\alpha\beta\gamma}\phi'_\alpha\phi'_\beta\phi'_\gamma \quad (3)$$

where $\lambda'_{\alpha\beta\gamma} \neq 0$ when $\alpha + \beta + \gamma = 2 \bmod n$. Thus we have proved that there exists a large number of generic (trivial) models where existence of a Z_n R-symmetry is sufficient to break SUSY via F-terms. One can also use this technique to other SUSY breaking models [5] with $U(1)$ R-symmetry.

One side comment about these models. Old and new sectors of any one of these models can communicate to each other through gauge interactions if we gauge some of the internal symmetries. For example, we can easily promote the fields ϕ and ϕ' to transform under adjoint representation and promote the field X to remain invariant under a gauge group, without forbidding any term of the old and new sectors. However, these models will then no longer be WZ models.

Now, we can ask whether it is possible to construct generic F-term SUSY breaking models with the following characteristics: (a) there is no $U(1)$ R-symmetry in the superpotential; (b) the superpotential cannot be subdivided into two disjoint sectors/parts. In the rest of the paper, we show examples of models with all these characteristics.

3 A non-trivial example with some variations

We consider a renormalizable WZ model with Z_{26} R-symmetry and $R_d(\theta) = 25$. We also consider that other than X , there are $\phi_3, \phi_6, \phi_8, \phi_9, \phi_{11}, \phi_{12}$ and ϕ_{13} fields in the theory. So we will have the following generic superpotential.

$$W_1 = fX + M_{11,13}\phi_{11}\phi_{13} + \frac{1}{2}M_{12,12}\phi_{12}^2 + \frac{1}{2}N_{13,13}X\phi_{13}^2 + \lambda_{3,8,13}\phi_3\phi_8\phi_{13} + \lambda_{3,9,12}\phi_3\phi_9\phi_{12} + \frac{1}{2}\lambda_{6,6,12}\phi_6^2\phi_{12} + \frac{1}{2}\lambda_{6,9,9}\phi_6\phi_9^2 + \frac{1}{6}\lambda_{8,8,8}\phi_8^3 \quad (4)$$

where, without loss of generality we can take all the parameters, except $\lambda_{8,8,8}$, to be real and positive. The above superpotential does not have a $U(1)$ R-symmetry. With $\lambda_{3,9,12} = 0$, there is a $U(1)$ R-symmetry with the following R-charge assignments.

X	ϕ_3	ϕ_6	ϕ_8	ϕ_9	ϕ_{11}	ϕ_{12}	ϕ_{13}
2	$\frac{4}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	2	1	0

(5)

But then the superpotential is not generic. For $\lambda_{3,9,12} \neq 0$, this $U(1)$ R-symmetry is explicitly broken.

There is spontaneous SUSY breaking. This can be easily realized by observing the following F-terms,

$$-F_X^* = f + \frac{1}{2}N_{13,13}\phi_{13}^2 \quad (6)$$

$$-F_{\phi_{11}}^* = M_{11,13}\phi_{13}. \quad (7)$$

Notice that vacuum expectation values (VEVs) of F_X and $F_{\phi_{11}}$ terms cannot be simultaneously zero.

We can vanish all other F-terms for any value of $\phi_{13}^{(0)}$ (VEV of ϕ_{13}) by choosing appropriate VEVs of other fields. Now minimum of scalar potential depends on $\phi_{13}^{(0)}$. Like the O'Raifeartaigh model [3] we have two cases, (a) for $y = \frac{fN_{13,13}}{M_{11,13}^2} < 1$, minimum is at $\phi_{13}^{(0)} = 0$, whereas (b) for $y > 1$, minimum is at $\phi_{13}^{(0)} = \pm i \frac{M_{11,13}}{N_{13,13}} \sqrt{2(y-1)}$.

Hence we have a vacuum where supersymmetry as well as discrete R-symmetry get spontaneously broken.

Tree level scalar potentials of SUSY breaking often have flat directions [7, 8]. For example, for the case of $y < 1$, minimum of tree level potential is independent of $X^{(0)}$ and $\phi_3^{(0)}$. So, it is necessary to calculate 1-loop correction to check whether these flat directions are lifted or not. One loop correction is given by Coleman-Weinberg (CW) [9] potential,

$$V_{CW} = \frac{1}{64\pi^2} \left(\text{tr} \left(M_B^4 \log \frac{M_B^2}{\Lambda_{\text{cutoff}}^2} \right) - \text{tr} \left(M_F^4 \log \frac{M_F^2}{\Lambda_{\text{cutoff}}^2} \right) \right), \quad (8)$$

where M_B and M_F are mass matrices for scalar and fermion fields. Non-zero eigenvalues λ^F and λ^B of M_F^2 and M_B^2 respectively for $y < 1$ are follows.

$$\begin{aligned} \lambda_{1,\eta}^F &= \lambda_{2,\eta}^F = \frac{1}{2} \left(M_{12,12}^2 + 2\lambda_{3,9,12} |\phi_3^{(0)}|^2 + \eta M_{12,12} \sqrt{M_{12,12}^2 + 4\lambda_{3,9,12}^2 |\phi_3^{(0)}|^2} \right) \\ \lambda_{3,\eta}^F &= \lambda_{4,\eta}^F = \frac{1}{2} \left(2M_{11,13}^2 + N_{13,13}^2 |X^{(0)}|^2 + 2\lambda_{3,8,13}^2 |\phi_3^{(0)}|^2 \right. \\ &\quad \left. + \eta N_{13,13} |X^{(0)}| \sqrt{4M_{11,13}^2 + N_{13,13}^2 |X^{(0)}|^2 + 4\lambda_{3,8,13}^2 |\phi_3^{(0)}|^2} \right) \\ \lambda_{1,\eta}^B &= \lambda_{2,\eta}^B = \lambda_{1,\eta}^F \\ \lambda_{3,\eta_1,\eta_2}^B &= \frac{1}{2} \left(\eta_2 f N_{13,13} + 2M_{11,13}^2 + N_{13,13}^2 |X^{(0)}|^2 + 2\lambda_{3,8,13}^2 |\phi_3^{(0)}|^2 + \eta_1 N_{13,13} \right. \\ &\quad \left. \sqrt{f^2 + |X^{(0)}|^2 (2\eta_2 f N_{13,13} + 4M_{11,13}^2 + N_{13,13}^2 |X^{(0)}|^2 + 4\lambda_{3,8,13}^2 |\phi_3^{(0)}|^2)} \right), \end{aligned}$$

where η , η_1 and η_2 denote ± 1 . Putting these eigenvalues to Eq. (8) and expanding V_{CW} about $X^{(0)} = \phi_3^{(0)} = 0$, we find

$$V_{CW} = \text{const.} + m_{X^{(0)}}^2 |X^{(0)}|^2 + m_{\phi_3^{(0)}}^2 |\phi_3^{(0)}|^2 + \mathcal{O}(|X^{(0)}|^4, |\phi_3^{(0)}|^4), \quad (9)$$

where

$$\begin{aligned} m_{X^{(0)}}^2 &= \frac{M_{11,13}^2 N_{13,13}^2}{32\pi^2} y^{-1} ((1+y)^2 \log(1+y) - (1-y)^2 \log(1-y) - 2y) \\ m_{\phi_3^{(0)}}^2 &= \frac{\lambda_{3,8,13}^2 M_{11,13}^2}{64\pi^2} ((1+y) \log(1+y) + (1-y) \log(1-y)). \end{aligned} \quad (10)$$

Constants $m_{X^{(0)}}^2$ and $m_{\phi_3^{(0)}}^2$ are positive and hence after addition of 1-loop correction, total scalar potential have a local minimum at $X^{(0)} = \phi_3^{(0)} = 0$.

We can get different variations of the above model by adding more ϕ fields in the theory. For example, we can add any number of fields from the list $\{\phi_{16}, \phi_{19}, \phi_{21}, \phi_{22}, \phi_{25}\}$. In this way we get 31 more models. If we add all the fields from the list, then the superpotential takes the following form

$$\begin{aligned} W &= W_1 + M_{3,21} \phi_3 \phi_{21} + M_{8,16} \phi_8 \phi_{16} + \frac{1}{2} M_{25,25} \phi_{25}^2 + \lambda_{3,22,25} \phi_3 \phi_{22} \phi_{25} \\ &\quad + \lambda_{6,19,25} \phi_6 \phi_{19} \phi_{25} + \frac{1}{2} \lambda_{6,22,22} \phi_6 \phi_{22}^2 + \frac{1}{2} \lambda_{8,21,21} \phi_8 \phi_{21}^2 + \lambda_{9,16,25} \phi_9 \phi_{16} \phi_{25} \\ &\quad + \lambda_{9,19,22} \phi_9 \phi_{19} \phi_{22} + \lambda_{12,13,25} \phi_{12} \phi_{13} \phi_{25} + \lambda_{12,16,22} \phi_{12} \phi_{16} \phi_{22} \\ &\quad + \frac{1}{2} \lambda_{12,19,19} \phi_{12} \phi_{19}^2 + \lambda_{13,16,25} \phi_{13} \phi_{16} \phi_{25}. \end{aligned} \quad (11)$$

Note that addition of these fields do not change F_X and $F_{\phi_{13}}$ and hence there is SUSY breaking.

4 Three series of models

In this section we are going to show that there exists a large number of non-trivial models where existence of a discrete R-symmetry is sufficient to break supersymmetry. We consider the superpotentials are of the following form with a Z_{6k+2q} R-symmetry and $R_d(\theta) = 6k + q$.

$$W = X(f + \frac{1}{2}N_{3k+q,3k+q}\phi_{3k+q}^2) + M_{3k-q,3k+q}\phi_{3k-q}\phi_{3k+q} + m(\phi) + \lambda(\phi), \quad (12)$$

where k, q are natural numbers, $\lambda(\phi)$ contains cubic terms which are independent of ϕ_{3k-q} , and $m(\phi)$ denotes quadratic terms for ϕ fields other than $\phi_{3k\pm q}$ fields. Note that the superpotentials of the previous section are of the above form with $k = 4$ and $q = 1$. Due to this form of superpotentials, SUSY gets spontaneously broken. One can show this by observing the following F-terms

$$\begin{aligned} -F_X^* &= f + \frac{1}{2}N_{3k+q}\phi_{3k+q}^2 \\ -F_{\phi_{3k-q}}^* &= M_{3k-q,3k+q}\phi_{3k+q}. \end{aligned} \quad (13)$$

Above equations are the same as Eq. (6) and (7) for $k = 4$ and $q = 1$.

Series I

In this series of models k is multiple of four and $q = 1$ i.e. superpotentials have a Z_{6k+2} R-symmetry with $R_d(\theta) = 6k + 1$. Field content of this series for any k is given below.

$$\left\{ X, \phi_k, \phi_{2k-1}, \phi_{2k}, \phi_{2k+2}, \phi_{3k-1}, \phi_{3k}, \phi_{3k+1}, \phi_{2k\pm 4i} \left(i = 1, 2, \dots, \frac{k}{4} - 1 \right) \right\}. \quad (14)$$

Note that the model with $k = 4$ have the same R-symmetry and $R_d(\theta)$ as the models given in the previous section. But this model is different from those models because it has different field content.

To show that there is no U(1) R-symmetry in this series of models, we use method of contradiction. If there were a U(1) R-symmetry in any model, existence of terms $X\phi_{3k+1}^2$, ϕ_{3k}^2 , and ϕ_{2k}^3 would imply that $R(\phi_{3k+1}) = 0$, $R(\phi_{3k}) = 1$ and $R(\phi_{2k}) = \frac{2}{3}$. From the terms $\phi_k\phi_{2k}\phi_{3k}$, $\phi_k\phi_{2k-1}\phi_{3k+1}$ and $\phi_{2k-1}^2\phi_{2k+2}$, we could conclude $R(\phi_k) = \frac{1}{3}$, $R(\phi_{2k-1}) = \frac{5}{3}$ and $R(\phi_{2k+2}) = -\frac{4}{3}$. Similarly we could construct a R-charge assignment chain for other fields as shown in Fig. 1. Now, for $k = 4$ we have $2k-4 = k$. But according to the chain (at the point A) $R(\phi_{2k-4}) = \frac{14}{3} \neq R(\phi_k)$. Hence the superpotential for $k = 4$ do not have a U(1) R-symmetry. Moving down the chain, one can easily show that there is no U(1) R-symmetry in any model of this series.

We are now going to prove that all the generic and renormalizable superpotentials of this series of models are of the form as given in Eq. (12).

There is no term quadratic in X in the superpotentials because field content for any k does not contain the field ϕ_2 . Also, cubic term in X is not allowed by discrete R-symmetries. Similarly, one can show that the terms $\phi_{3k-1}^2\phi_2$ and ϕ_{3k-1}^3 are also not allowed.

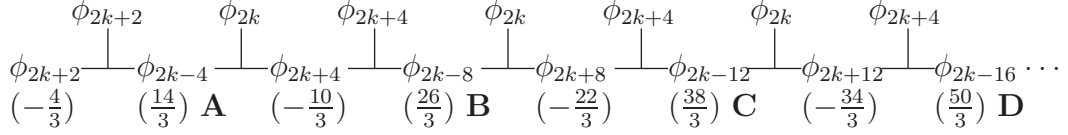


Figure 1: Diagram representing some cubic terms in superpotentials for series I. Values inside parentheses represent U(1) R-charges. For any field ϕ_{2k+4i} , U(1) R-charge is $\frac{-12i+2}{3}$ where i is an integer. The chain will be truncated at **A**, **B**, **C**, ... for $k = 4, 8, 12, \dots$

For a cubic term of the form $\phi_i \phi_j \phi_{3k-1}$ to exist, we need $i + j = 3k + 1$. Without loss of generality, we can take $i \leq j$. From field content given in Eq. (14), we find $i \geq k$ and hence $j \leq 2k + 1$. Also one of them must be odd since their sum is odd. The only field having odd discrete R-charge in this range is ϕ_{2k-1} . But field content of any model does not contain the field ϕ_{k+2} and so, λ -terms are independent of the field ϕ_{3k-1} . There is only one cubic term containing X , $\frac{1}{2} N_{3k+1, 3k+1} X \phi_{3k+1}^2$, because discrete R-charges of the ϕ fields lie between k and $3k + 1$. Thus superpotentials of this series are of form as given in Eq. (12) and hence there is F-term SUSY breaking.

Let's now give superpotential for $k = 4$.

$$\begin{aligned}
W = & fX + M_{11,13} \phi_{11} \phi_{13} + \frac{1}{2} M_{12,12} \phi_{12}^2 + \frac{1}{2} N_{13,13} X \phi_{13}^2 + \lambda_{4,7,13} \phi_4 \phi_7 \phi_{13} \\
& + \lambda_{4,8,12} \phi_4 \phi_8 \phi_{12} + \frac{1}{2} \lambda_{4,10,10} \phi_4 \phi_{10}^2 + \frac{1}{2} \lambda_{7,7,10} \phi_7^2 \phi_{10} + \frac{1}{6} \lambda_{8,8,8} \phi_8^3.
\end{aligned} \tag{15}$$

One can explicitly verify that the above superpotential does not have a U(1) R-symmetry yet there is F-term SUSY breaking.

Series II

Superpotentials of this series have Z_{6k+4} R-symmetry where k is multiple of 6 with starting value 12. Discrete R-charge of superspace co-ordinate θ is $6k + 2$ or $q = 2$. Field content for any k is given below.

$$\left\{ X, \phi_k, \phi_{2k-6}, \phi_{2k-2}, \phi_{2k-1}, \phi_{2k}, \phi_{2k+1}, \phi_{2k+3}, \phi_{2k+6}, \phi_{3k-2}, \right. \\
\left. \phi_{3k}, \phi_{3k+2}, \phi_{2k \pm 6i} \left(i = 2, 3, \dots, \frac{k}{6} - 1 \right) \right\} \tag{16}$$

To show that there is no U(1) R-symmetry, we have taken same the strategy as of the earlier case. Let's first tabulate U(1) R-charges of first twelve fields from the above list.

X	ϕ_k	ϕ_{2k-6}	ϕ_{2k-2}	ϕ_{2k-1}	ϕ_{2k}	ϕ_{2k+1}	ϕ_{2k+3}	ϕ_{2k+6}	ϕ_{3k-2}	ϕ_{3k}	ϕ_{3k+2}
2	$\frac{1}{3}$	$\frac{11}{3}$	$\frac{5}{3}$	$\frac{7}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	$-\frac{5}{6}$	$-\frac{7}{3}$	2	1	0

(17)

These fields are common to all models of this series. U(1) R-charges for the fields $\phi_{3k+2}, \phi_{3k}, \phi_{3k-2}, \phi_{2k-2}, \phi_{2k}$ and ϕ_k can be derived easily. We obtained R-charges for $\phi_{2k+1}, \phi_{2k-1}, \phi_{2k+3}, \phi_{2k-6}, \phi_{2k+6}$ from the terms $\phi_{2k-2} \phi_{2k+1}^2$,

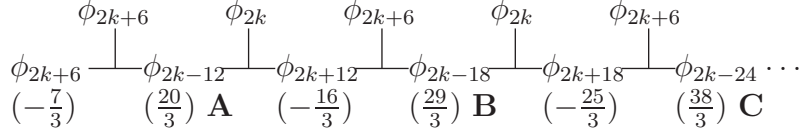


Figure 2: Diagram representing some cubic terms in superpotentials for series II. Values inside parentheses represent U(1) R-charges. For any field ϕ_{2k+6i} , discrete R-charge is $\frac{-9i+2}{3}$ where i is an integer. The chain will truncate at **A**, **B**, **C**, ... for $k = 12, 18, 24, \dots$

$\phi_{2k+1}\phi_{2k}\phi_{2k-1}$, $\phi_{2k-2}\phi_{2k-1}\phi_{2k+3}$, $\phi_{2k+3}^2\phi_{2k-6}$ and $\phi_{2k-6}\phi_{2k}\phi_{2k+6}$ respectively. U(1) R-charge assignment chain for rest of the fields is given in the Fig. 2. From these given information, one can easily show that there is no U(1) R-symmetry in any model of this series.

There will be a $\lambda_{i,j,3k-2}$ -term only if $i + j = 3k + 2$. Without loss of generality, we can take $i \leq j$. Minimum value for i is k . As there is no field with R-charge $2k + 2$, i cannot be equals to k . Next higher value of discrete R-charge is $k + 6$ and hence $k + 6 \leq i \leq j < 2k - 2$. In this range discrete R-charges of the fields are multiple of 6. As $3k + 2$ is not multiple of 6, there cannot a λ -term for ϕ_{3k-2} . So, superpotentials of this series also of the form as given in Eq. (12) and which in turn guarantee spontaneous breakdown of SUSY.

Let's give first model of this series.

$$\begin{aligned}
W = & fX + M_{34,38}\phi_{34}\phi_{38} + \frac{1}{2}M_{36,36}\phi_{36}^2 + \frac{1}{2}XN_{38,38}\phi_{38}^2 + \lambda_{12,22,38}\phi_{12}\phi_{22}\phi_{38} \\
& + \lambda_{12,24,36}\phi_{12}\phi_{24}\phi_{36} + \frac{1}{2}\lambda_{12,30,30}\phi_{12}\phi_{30}^2 + \lambda_{22,23,27}\phi_{22}\phi_{23}\phi_{27} + \frac{1}{2}\lambda_{22,25,25}\phi_{22}\phi_{25}^2 \\
& + \lambda_{23,24,25}\phi_{23}\phi_{24}\phi_{25} + \frac{1}{6}\lambda_{24,24,24}\phi_{24}^3 + \lambda_{18,24,30}\phi_{18}\phi_{24}\phi_{30} + \frac{1}{2}\lambda_{18,27,27}\phi_{18}\phi_{27}^2 \\
& + \frac{1}{2}\lambda_{18,18,36}\phi_{18}^2\phi_{36}
\end{aligned} \tag{18}$$

One can explicitly verify that there is no U(1) R-symmetry and SUSY is spontaneously broken.

Series III

Superpotentials of this series have Z_{6k+6} R-symmetry with $R_d(\theta) = 6k + 3$ and $k = 8, 10, 12, 14, \dots$. Field content for any k is given below.

$$\left\{ X, \phi_k, \phi_{2k-4}, \phi_{2k}, \phi_{2k+2}, \phi_{2k+8}, \phi_{3k-3}, \phi_{3k}, \phi_{3k+3}, \right. \\
\left. \phi_{4k+2}, \phi_{6k+2}, \phi_{6k+3}, \phi_{6k+4}, \phi_{6k+5}, \phi_{2k \pm 2i} \left(i = 5, 6, \dots, \frac{k}{2} - 1 \right) \right\} \tag{19}$$

If we demand that the superpotentials have an U(1) R-symmetry, then we will have a table (Eq. (20)) and a chain (Fig. (3)) of R-charge assignments. From these inputs one can conclude that there is no U(1) R-symmetry in any model of this series.

X	ϕ_k	ϕ_{2k}	ϕ_{2k+2}	ϕ_{3k-3}	ϕ_{3k}	ϕ_{3k+3}	ϕ_{4k+2}	ϕ_{6k+2}	ϕ_{6k+3}	ϕ_{6k+4}	ϕ_{6k+5}
2	$\frac{1}{3}$	$\frac{2}{3}$	0	2	1	0	$\frac{2}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$

(20)

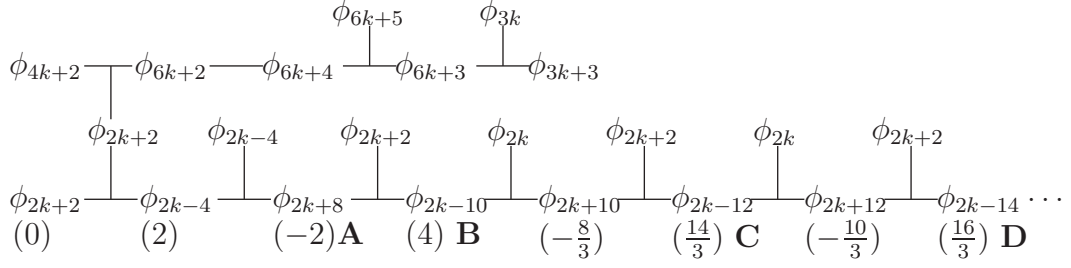


Figure 3: Diagram representing some cubic terms in superpotentials for $n = 6k + 6$. Values inside parentheses represent U(1) R-charges. For any field ϕ_{2k+2i} , U(1) R-charge is $\frac{-2i+2}{3}$ where i is an integer. Most of the fields from Eq. (20) are also added to the chain so that one can easily determine the U(1) R-charge easily.

There will be a λ -term for ϕ_{3k-3} only if $i + j = 3k + 3 \bmod(6k + 6)$ where i, j are discrete R-charges of fields coupled to it. Thus

$$i + j = 3k + 3 \text{ or } i + j = 9k + 9 \quad (21)$$

For first case $k \leq i, j \leq 2k + 3$. As there is no field with odd discrete R-charge in this range, this possibility is ruled out. One can check that second possibility is also ruled out. Hence superpotentials of this series are also of the form given in Eq. (12) and there are F-term SUSY breaking.

Let's give first model of this series, i.e. for $k = 8$, so that one can verify non-existence of a U(1) R-symmetry and spontaneous breakdown of SUSY.

$$\begin{aligned} W = & fX + M_{21,27}\phi_{21}\phi_{27} + \frac{1}{2}M_{24,24}\phi_{24}^2 + M_{50,52}\phi_{50}\phi_{52} + \frac{1}{2}M_{51,51}\phi_{51}^2 \\ & + \frac{1}{2}XN_{27,27}\phi_{27}^2 + \lambda_{8,16,24}\phi_8\phi_{16}\phi_{24} + \frac{1}{2}\lambda_{12,12,24}\phi_{12}^2\phi_{24} + \frac{1}{2}\lambda_{12,18,18}\phi_{12}\phi_{18}^2 \\ & + \frac{1}{6}\lambda_{16,16,16}\phi_{16}^3 + \lambda_{16,34,52}\phi_{16}\phi_{34}\phi_{52} + \lambda_{18,34,50}\phi_{18}\phi_{34}\phi_{50} + \lambda_{24,27,51}\phi_{24}\phi_{27}\phi_{51} \\ & + \frac{1}{6}\lambda_{34,34,34}\phi_{34}^3 + \frac{1}{2}\lambda_{50,53,53}\phi_{50}\phi_{53}^2 + \lambda_{51,52,53}\phi_{51}\phi_{52}\phi_{53} + \frac{1}{6}\lambda_{52,52,52}\phi_{52}^3. \end{aligned} \quad (22)$$

In the above we have given only three series of models. However one can construct many series of such models.

5 Conclusions

We have shown that there exists a large number of generic and renormalizable Wess-Zumino models where existence of a Z_n R-symmetry is sufficient to break SUSY spontaneously. And it is well known that if there is no R-symmetry in WZ models with canonical Kähler potential, then global minima always preserve SUSY. So, existence of a Z_n R-symmetry in a generic and renormalizable Wess-Zumino model is a necessary condition for F-term SUSY breaking. However, for even n with $R_d(\theta) = \frac{n}{2}$, one cannot have models of SUSY breaking because for these cases superpotentials as a whole transform trivially and terms which are allowed or forbidden by these R-symmetries can always be reproduced by some internal symmetries.

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